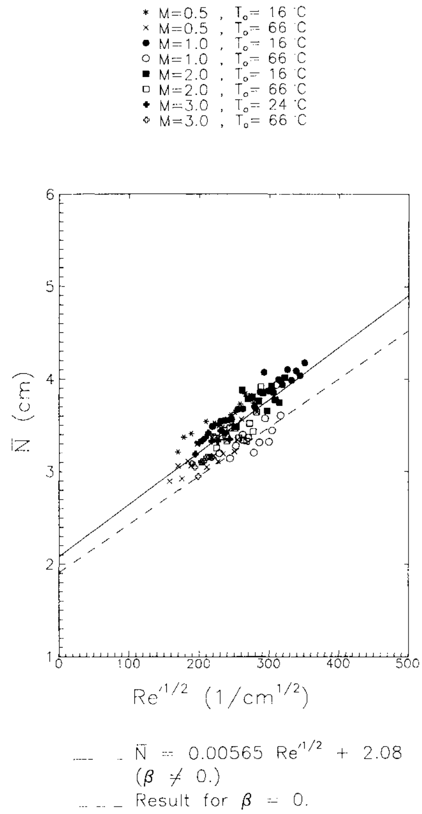

Fig. 4 Heat-transfer characteristics of typical probe; *R-W* data fit with a second-degree polynomial.

Mach number. If the derivatives $\partial\eta/\partial Re'$ and $\partial\tilde{N}/\partial M$ can indeed be neglected while applying these probes to turbulence measurements, then the formulas needed to translate the probe alternating current signals to turbulence intensities[10] can be visibly simplified.

References

[1]Laderman, A. J., and Demetriades, A., "Turbulent Shear Stresses in Compressible Boundary Layers," *AIAA Journal*, Vol. 17, No. 7, 1979, p. 736.

[2]Demetriades, A., "Turbulence Measurements in an Axisymmetric Compressible Wake," *Physics of Fluids*, Vol. 11, No. 9, 1968, p. 1841.

[3]Demetriades, A., "New Experiments on Hypersonic Boundary Layer Stability Including Wall Temperature Effects," Heat Transfer and Fluid Mechanics Inst., Stanford Univ. Press, Stanford, CA, 1978, p. 39.

[4]Bellhouse, B. J., and Bellhouse, F. H., "Thin-Film Gauges for Measurement of Velocity or Skin Friction in Air, Water, or Blood," *Journal of Physics E.: Scientific Instruments*, Vol. 1, 1968, Series 2, pp. 1211–1213.

[5]Demetriades, A., Munger, C. O., and Anders, S. G., "Hot Film Anemometer Probes for High-Temperature Hypersonic Research," Montana State Univ., Bozeman, MT, MSU SWT TR 89 01, March 1989.

[6]Hinze, J. O., *Turbulence*, 1st ed., McGraw-Hill, New York, 1959, p. 78.

[7]Hertel, P. S., "Heat Transfer Characteristics of Small Configuration Hot-Wires in Low Reynolds Number Subsonic and Supersonic Air Flows," Master's Thesis, Montana State Univ., Bozeman, MT, March 1985.

[8]Dewey, C. F., Jr., "A Correlation of Convective Heat Transfer and Recovery Temperature Data for Cylinders in Compressible Flow," *International Journal of Heat and Mass Transfer*, Vol. 8, Feb. 1965, pp. 245–252.

[9]Seiner, J. M., "The Wedge Hot-Film Anemometer in Supersonic Flow," NASA TP-2134, May 1983, p. 56.

[10]Morkovin, M. V., "Fluctuations and Hot-Wire Aneomometry in Compressible Flows," AGARDograph 24, Paris, France, 1956.

Bending of Thick Angle-Ply Bimodular Composite Plates

Faramarz Gordaninejad*
University of Nevada, Reno, Reno, Nevada 89557

Introduction

A BIMODULAR (or bimodulus) material is one which behaves differently under tension and compression loads, and its nonlinear stress-strain curve is approximated by two straight lines: one in tension and the other in compression. Many fiber-reinforced composites, such as graphite/epoxy and tire-cord rubber exhibit this behavior. Within the last two decades, numerous investigations have been performed for isotropic and orthotropic beam, plate, and shell structures constructed from bimodular materials. In the following, only those studies related to the analysis of bimodular composite plates are briefly addressed. For a comprehensive review of the literature, readers are referred to a study by Bert and Reddy.[1]

Kamiya[2,3] studied the large deformation of thin, isotropic, clamped circular and rectangular plates using the finite-difference and Galerkin methods, respectively. Singh et al.[4] reported results for bending of a thin, isotropic bimodular plate as applied to rock mechanics. They determined the position of the neutral surface of the plate by setting the total in-plane shear force equal to zero. Bert and Kincannon,[5] Reddy and Choa,[6] Reddy and Bert,[7] and Bert et al.[8] extended Kamiya's work to the analysis of thick, orthotropic laminated bimodular plates. They utilized the fiber-governed compliance theory introduced by Bert[9] to determine the position of neutral surface. Later, they extended their studies to large deflection,[10] thermal stress,[11] vibration,[12] and transient response[13] of orthotropic, laminated, bimodular composite plates. Doong and Chen[14] addressed the problem of vibration of thick, orthotropic plates under combined bending and extensional stresses. Gordaninejad[15] studied the effect of shear deformation on bending of orthotropic, bimodular composite plates by implementing Reddy's higher-order consistent shear deformation theory.[16] Furthermore, he developed a finite element model for the analysis of single-layer, anisotropic, bimodular composite plates.[17,18]

In the present study, the model in Ref. 17 is extended to analyze laminated anisotropic bimodular composite plates. The shear-deformation effect is included by employing the plate theory of Yang et al.[19] The position of the neutral surface is determined by using the fiber-governed compliance theory.[9] Because of the dependency of the neutral surface on the generalized displacements and angle of fiber orientation, no closed-form solution can be obtained even for the simplest loading cases and boundary conditions. Therefore, the displacement finite element method is utilized to solve the problem.

Theory and Formulation

Consider a two-layer, angle-ply, antisymmetric, rectangular plate of constant thickness h with fibers oriented at arbitrary angles $(\theta/-\theta)$. A Cartesian coordinate system is set at the center of the plate's midsurface with the z axis normal to the midsurface. The total displacements U and V, presented in Eq. (1), vary linearly and transverse deflection W remains constant through the thickness of the plate:

$$U = u + z\psi_x; \qquad V = v + z\psi_y; \qquad W = w \tag{1}$$

Received March 31, 1989; revision received Nov. 27, 1989.

*Associate Professor, Mechanical Engineering Department.

Here, u, v, and w are midsurface in-plane axial, in-plane transverse, and out-of-plane transverse displacements, respectively. The ψ_x and ψ_y are bending slopes in the x and y directions, respectively. In the present analysis, a small-strain approximation is considered; therefore, the strain-displacement relations are reduced to

$$\epsilon_x = u_{,x} + z\psi_{x,x}; \qquad \epsilon_y = v_{,y} + z\psi_{y,y}$$

$$\epsilon_{xy} = u_{,y} + v_{,x} + z(\psi_{x,y} + \psi_{y,x})$$

$$\epsilon_{yz} = w_{,y} + \psi_y; \qquad \epsilon_{xz} = w_{,x} + \psi_x \tag{2}$$

Ignoring the normal stress in the z direction, the linearly elastic constitutive equations, for an orthotropic layer in the coordinate system rotated by an angle θ away from the principal material coordinate in the plate of the lamina, become

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11_p} & \bar{Q}_{12_p} & \bar{Q}_{16p} \\ & \bar{Q}_{22_p} & \bar{Q}_{26_p} \\ \text{symm.} & & \bar{Q}_{66_p} \end{bmatrix}_k \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{44_p} & \bar{Q}_{45_p} \\ \bar{Q}_{45_p} & \bar{Q}_{55_p} \end{bmatrix}_k \begin{Bmatrix} \epsilon_{yz} \\ \epsilon_{xz} \end{Bmatrix} \tag{4}$$

where $\bar{Q}_{ij_p}$ are the transformed plane-stress-reduced stiffnesses, σ_x and σ_y are normal stresses, and σ_{xy}, σ_{yz}, and σ_{xz} are shear stresses. The subscript p refers to tension (t) or compression (c) regions, and subscript k represents the kth layer. The fiber-governed compliance theory has been shown to give better agreement with experimental results[20], therefore, it is being adopted here to define the transformed plane-stress-reduced stiffnesses of the plate, as follows:

$$Q_{ij_{pk}} = \begin{cases} \bar{Q}_{ij_{tk}} & \text{if} \quad \epsilon_f \geq 0 \\ \bar{Q}_{ij_{ck}} & \text{if} \quad \epsilon_f < 0 \end{cases} \tag{5}$$

where ϵ_f is the strain in the fiber direction.

The expression for the total potential energy associated with the bending of a thick, anisotropic plate under a distributed pressure q is given by

$$\begin{aligned} I = {} & \frac{1}{2}\int\!\!\int\!\!\int (\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_{yz}\epsilon_{yz} + \sigma_{xz}\epsilon_{xz} + \sigma_{xy}\epsilon_{xy})\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ & + \int\!\!\int qw\,\mathrm{d}x\,\mathrm{d}y = \frac{1}{2}\int\!\!\int \{A_{11}u_{,x}^2 + 2A_{12}u_{,x}u_{,y} + A_{22}v_{,y}^2 \\ & + 2A_{16}u_{,x}(u_{,y} + v_{,x}) + 2A_{26}v_{,y}(u_{,y} + v_{,x}) \\ & + A_{66}(u_{,y} + v_{,x})^2 + 2B_{11}u_{,x}\psi_{x,x} \\ & + 2B_{12}(u_{,x}\psi_{y,y} + v_{,y}\psi_{x,x}) + 2B_{16}[u_{,y}\psi_{x,x} \\ & + u_{,x}(\psi_{x,y} + \psi_{y,x})] + 2B_{26}[u_{,y}\psi_{y,y} + v_{,y}(\psi_{x,y} + \psi_{y,x})] \\ & + 2B_{66}(u_{,y} + v_{,x})(\psi_{x,y} + \psi_{y,x})D_{11}\psi_{x,x}^2 + 2D_{12}\psi_{x,x}\psi_{y,y} \\ & + D_{22}\psi_{y,y}^2 + 2D_{16}\psi_{x,x}(\psi_{x,y} + \psi_{y,x}) + 2D_{26}\psi_{y,y}(\psi_{x,y} + \psi_{y,x}) \\ & + D_{66}(\psi_{x,y} + \psi_{y,x})^2 + A_{55}(w_{,x} + \psi_x)^2 \\ & + 2A_{45}(w_{,y} + \psi_y)(w_{,x} + \psi_x) \\ & + A_{44}(w_{,y} + \psi_y)^2 + qw\}\,\mathrm{d}x\,\mathrm{d}y \end{aligned} \tag{6}$$

where stiffnesses associated with stretching (A_{ij}; i,j = 1, 2, 6), bending-stretching (B_{ij}), bending (D_{ij}), and shear (A_{ij}; i,j = 4, 5) for the kth layer, rectangular plate with uniform thickness can be expressed as

$$\begin{aligned} (A_{ij}, B_{ij}, D_{ij}) = {} & \sum_{k=1}^{n}\left[\int_{z_{(k-1)}}^{(z_n)_k} \bar{Q}_{ij_{pk}}(1, z, z^2)\,\mathrm{d}z \right. \\ & \left. + \int_{(z_n)_k}^{z_k} \bar{Q}_{ij_{pk}}(1, z, z^2)\,\mathrm{d}z\right]; \qquad i,j = 1,2,6 \end{aligned} \tag{7}$$

$$A_{ij} = K\sum_{k=1}^{n}\int_{z_{(k-1)}}^{(z_n)_k} \bar{Q}_{ij_{pp}}\,\mathrm{d}z; \qquad i, j = 4,5 \tag{8}$$

where n is the number of layers, z_n is the position of neutral surface, and K is the shear correction coefficient. No shear bimodular action is considered in this analysis. This is a reasonable assumption for the materials considered (see Table 1). The position of the neutral surface is obtained by setting the strain along the fibers equal to zero.

$$0 = \epsilon_f = \epsilon_x \cos^2\theta_k + \epsilon_{xy}\sin\theta_k\cos\theta_k + \epsilon_y\sin^2\theta_k \tag{9}$$

By combining Eqs. (2) and (9), one has

$$(z_n)_k = \frac{u_{,x}\cos^2\theta_k + (u_{,y} + v_{,x})\sin\theta_k\cos\theta_k + v_{,y}\sin\theta_k}{\psi_{x,x}\cos^2\theta_k + (\psi_{x,y} + \psi_{y,x})\sin\theta_k\cos\theta_k + \psi_{y,y}\sin^2\theta_k} \tag{10}$$

Equation (10) is highly coupled and nonlinear, and there is no guarantee for existence of the solution within the domain of the kth layer. Therefore, only if a neutral surface exists in the kth lamina the above expression will be satisfied.

The finite element formulation is developed, over each element, by utilizing the following interpolation of variables

$$\Delta^e = \sum_{i=1}^{n}\Delta_i^e\phi_i^e \tag{11}$$

where $\Delta_i^e = \{u^e, v^e, w^e, \psi_x^e, \psi_y^e\}_i^T$ is the nodal generalized displacements, and superscript e represents an element. For a typical element, the substitution of Eq. (11) into the first variation of Eq. (6) yields

$$[K]^e\{\Delta\}^e = \{F\}^e \tag{12}$$

where the elements of stiffness matrix $[K]^e$ and generalized force vector $\{F\}^e$ are given in Ref. 17.

Numerical Results and Discussion

In the following, the numerical results are presented for thick, two-layer, angle-ply, rectangular, bimodular composite plates under a uniformly distributed load. Two materials, one with a low (graphite/epoxy) and another with a high (Aramid/rubber) bimodularity ratio, are examined. Both materials are widely used in the aerospace and tire industries. The material properties are listed in Table 1. The shear correction coefficient is assumed to be 5/6.

Table 1 Material properties of graphite/epoxy and Aramid/rubber

	Graphite/epoxy		Aramid/rubber	
Property	Tensile	Compressive	Tensile	Compressive
E_{11}, Gpa	165.52	151.72	3.58	0.012
E_{22},Gpa	8.28	7.59	0.00909	0.012
$G_{12} = G_{13}$,Gpa	2.59	2.59	0.0037	0.0037
G_{23},Gpa	2.59	2.59	0.0029	0.00499
ν_{12}	0.25	0.25	0.416	0.205

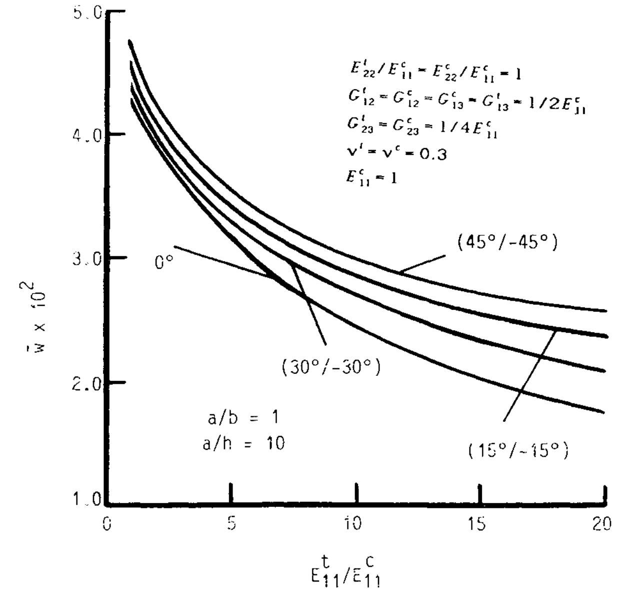

Fig. 1 Effects of bimodularity ratio (E^t_{11}/E^c_{11}) and fiber orientation.

Table 2 Effects of plate aspect ratio and angle of fiber orientation, $\bar{w} \times 10^2$

Material	a/b	$\theta = 0$ deg	$\theta = 15$ deg	$\theta = 30$ deg	$\theta = 45$ deg
	1.0	1.2289	1.4349	1.4711	1.4207
Graphite/	1.5	0.7791	0.8091	0.6819	0.5367
epoxy	2.0	0.4788	0.4612	0.3325	0.2399
	2.5	0.2817	0.2591	0.1739	0.1201
	1.0	0.6792	1.0269	1.4805	1.9497
Aramid/	1.5	0.3742	0.6169	0.5763	0.6387
rubber	2.0	0.2453	0.3488	0.3808	0.2778
	2.5	0.1700	0.2124	0.2064	0.1368

Table 3 Effects of different material properties and fiber orientation $\bar{w}^a \times 10^2$

Material	θ deg	Average[b]	Tensile	Compressive	Bimodular
	0	1.2846	1.3072	1.2618	1.2289
Graphite/	15	1.4991	1.5193	1.4785	1.4352
epoxy	30	1.5368	1.5514	1.5220	1.4717
	45	1.4840	1.4963	1.4718	1.4200
	0	0.5712	0.4454	5.4820	0.6791
Aramid/	15	0.6107	0.4343	5.3530	1.0269
rubber	30	0.4932	0.3580	5.0871	1.4805
	45	0.4553	0.3387	4.9663	1.9497

[a] $\bar{w} = wE_{ref}h^3/qa^4$, $E_{ref} = (E^c_{22})$ compressive; [b] $E_{avg} = (E^t + E^c)/2$.

A simply supported boundary condition, the same as Boundary Condition 1 in Ref. 21, is imposted on all four edges of the plate as follows:

$$v(-a/2,y) = w(-a/2,y) = \psi_y(-a/2,y) = 0$$

$$v(+a/2,y) = w(+a/2,y) = \psi_y(+a/2,y) = 0$$

$$u(x, -b/2) = w(x, -b/2) = \psi_x(x, -b/2) = 0$$

$$u(x, +b/2) = w(x, +b/2) = \psi_x(x, +b/2) = 0 \qquad (13)$$

The computation strategy to solve Eq. (12) is based on an iterative procedure. Since the location of the neutral surface is a function of the generalized displacements as well as the fiber orientation, the plate stiffness coefficients are not constant. Therefore, the initial values for z_n at the nodal points must be prescribed to obtain the displacement vector by combining Eqs. (7-9), and (12). Then, the new set of z_n is computed from Eq. (10). This procedure is repeated until a convergence criterion is satisfied. In this study Picard criterion is used. A finite element mesh of 4×4 (of nine-node isoparametric element) for the whole plate was considered. The computations were performed on an IBM 4381 computer using double precision arithmetic.

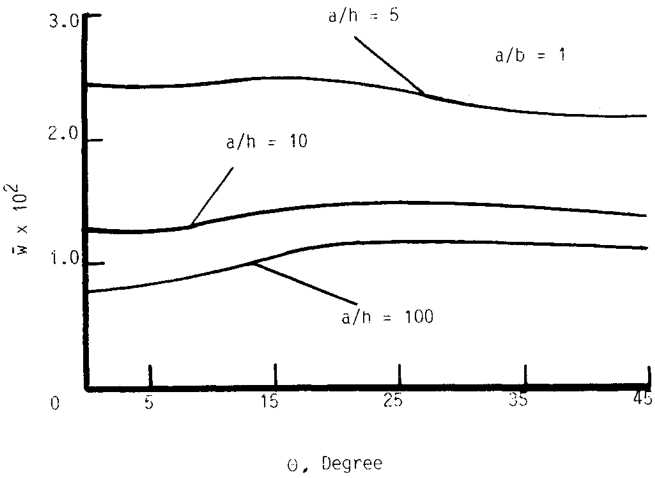

Fig. 2 Effects of shear deformation and fiber orientation on bending of graphite/epoxy plates.

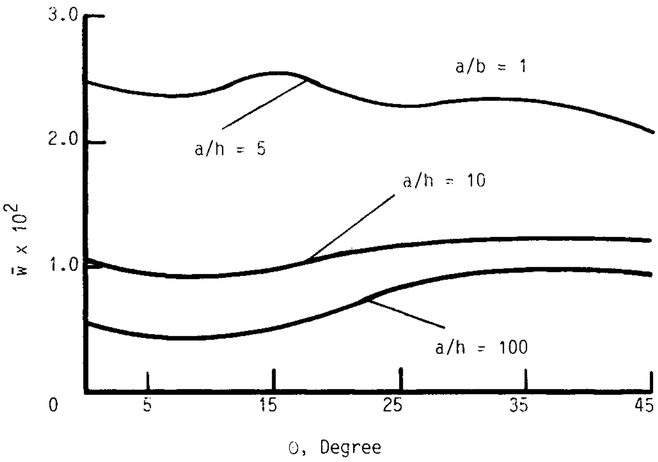

Fig. 3 Effects of shear deformation and fiber orientation on bending of Aramid/rubber plates.

The effects of plate aspect ratio (a/b) and fiber orientation θ on the maximum dimensionless transverse deflection $\bar{w} = wE_{22}h^3/qa^4$ of moderately thick ($a/h = 10$) plates are presented in Table 2. The dimensionless deflection decreases by increasing the aspect ratio of all θ. The variation of deflection for both graphite/expoxy and Aramid/rubber is highly nonlinear with respect to fiber orientation for a given a/b. The influence of bimodularity ratio E^+_{22}/E^c_{11} on the dimensionless tranverse deflection of moderately thick ($a/h = 10$) square plates ($a/b = 1$) for different fiber orientation is shown in Fig. 1. By increasing this ratio (where $E^c_{11} = 1$), the dimensionless deflection decreases nonlinearly for all θ.

The effect fiber orientation on the maximum nondimensionalized center deflection of thick ($a/h = 5$), moderately thick ($a/h = 10$), and thin ($a/h = 100$) graphite/expoxy and Aramid/rubber square plates is demonstrated in Figs. 2 and 3, respectively. First of all, the trend of the variation with respect to angle θ is generally the same for both materials in spite of large difference between their bimodularity ratios. This suggests that the effect of bimodularity on shear deformation is minimal. Second, the variation is more pronounced for thick plates than thin ones. For a side-to-thickness ratio of 5, maximum deflection occurs at 15 deg for graphite/epoxy and about 13 deg for Aramid/rubber. For moderately thick and thin plates, the maximum deflection shifts to about 27 deg for graphite/expoxy and 35 deg for Aramid/rubber.

Finally, to demonstrate the difference between the bimodular and "unimodular" assumptions, comparisons were made among the dimensionless center deflections of simply supported, square, two-layer, angle-ply plates under uniformly distributed load using average, tensile, compressive, and bimodular properties (see Table 3). The results from the two models, namely bimodular and unimodular, differ slightly from each other for graphite/epoxy (up to about 6%). However, for Aramid/rubber, which has a bimodularity ratio of about 300, the difference increases up to 88%.

Concluding Remarks

In this study, a displacement finite element model is developed for bending of thick, rectangular, antisymmetric, angle-ply, fiber-reinforced, bimodular composite plates. By examining the results, it is concluded that the effects of plate aspect ratio, bimodularity ratio, fiber orientation, and shear deformation are considerable on the transverse deflection. In addition, the result shows that the influence of bimodularity ratio on shear deformation is minimal. Furthermore, for materials with a high bimodularity ratio, a unimodular assumption may result in a very large error in the displacement prediction.

References

[1]Bert, C. W., and Reddy, J. N. "Mechanics of Bimodular Composite Structures," *Mechanics of Composite Materials, Recent Advances*, Proceedings of the IUTAM Symposium, Pergamon, New York, 1983, pp. 323-337.

[2]Kamiya, N., "Large Deflection of a Different Modular Circular Plate," *ASME Journal of Engineering Matls. and Tech.*, Vol. 97H, 1976, pp. 52-56.

[3]Kamiya, N., "An Energy Method Applied to Large Elastic Deflection of a Thin Plate of Bimodulus Material," *Journal of Structural Mechanics*, Vol. 3, 1975, pp. 317-329.

[4]Singh, J. G., Upadhyay, P. C., and Saluja, S. S., "The Bending of Rock Plates," *International Journal of Rock Mechanics, Mining Science & Geomechanic Abstracts*, Vol. 17, 1980, pp. 372-381.

[5] Bert, C. W., and Kincannon, S. K., "Bending-Extensional Coupling in Elliptic Plates of Orthotropic Bimodulus Material," *Developments in Mechanics*, Vol. 10, *Proceedings of the 16th Midwestern Mechanics Conference*, Kansas State Univ., Manhattan, KS, 1979, pp. 7-11.

[6]Reddy, J. N., and Choa, W. C., "Finite-Element Analysis of Laminated Bimodulus Composite Material Plates," *Computers and Structures*, Vol. 12, 1980, pp. 245-251.

[7]Reddy, J. N., and Bert, C. W., "On the Behavior of Plates Laminated of Bimodulus Composite Material," Vol. 62, No. 2, 1981, pp. 213-219.

[8]Bert, C. W., Reddy, J. N., Reddy, V. S., and Choa, W. C., "Bending of Thick Rectangular Plates Laminated of Bimodulus Composite Materials," *AIAA Journal*, Vol. 19, 1981, pp. 1342-1349.

[9]Bert, C. W., "Models for Fibrous Composites with Different Properties in Tension and Compression," *ASME J. Engrg Matls. and Tech. Trans.*, Vol. 19, 1977, pp. 344-349.

[10]Reddy, J. N., and Choa, W. C., "Nonlinear Bending of Bimodular-Material Plates," *International Journal of Solid Structures*, Vol. 19, 1983, pp. 121-147.

[11]Reddy, J. N., Bert, C. W., Hsu, Y. C., and Reddy, V. S., "Thermal Bending of Thick Rectangular Plates of Bimodulus Composite Materials," *International Journal of Mechanical Engineering Sciences*, Vol. 22, 1980, pp. 297-304.

[12]Bert, C. W., Reddy, J. N., Choa, W. C., and Reddy, V. S., "Vibration of Thick Rectangular Plates of Bimodulus Composite Material," *Journal of Composite Materials*, Vol. 48, 1981, pp. 371-376.

[13]Reddy, J. N., "Transient Response of Laminated, Bimodular-Material, Composite Rectangular Plates," *Journal of Composite Materials*, Vol. 16, 1982, pp. 139-152.

[14]Doong, D. L., and Chen, L. W., "Vibration of a Bimodulus Thick Plate," American Society of Mechanical Engineers, New York, Paper 83-Det-29, 1983.

[15]Gordaninejad, F., "Effect of Shear Deformation on Bending of Bimodular Composite-Material Plates," *Journal of Composite Structures*, Vol. 12, 1989, pp. 161-170.

[16]Reddy, J. N., "A Simple Higher-Order Theory for Laminated Composite Plates," *Journal of Applied Mechanics*, Vol. 51, 1984, pp. 745-752.

[17]Gordaninejad, F., "A Finite-Element Model for the Analysis of Thick, Anisotropic, Bimodular, Fibrous-Composite Plates," *Computers and Structures*, Vol. 31, 1989, pp. 907-912.

[18]Gordaninejad, F., "Nonlinear Bending of Anisotropic Bimodular Composite-Material Plates," *Computers and Structures*, Vol. 33, No. 3, 1989, pp. 615-620.

[19]Yang, P. C., Norris, C. H., and Stavsky, Y., "Elastic Wave Propagation in Heterogeneous Plates," *International Journal of Solid Structures*, Vol. 2, 1966, pp. 665-684.

[20]Schmueser, D. W., "Nonlinear Stress-Strain and Strength Response of Axisymmetric Bimodulus Composite Material Shells," *AIAA Journal*, Vol. 21, 1982, pp. 1742-1747.

[21]Reddy, J. N., "A Note on Symmetry Consideration in the Transient Response of Unsymmetrically Laminated Anisotropic Plates," *International Journal for Numerical Methods in Engineering*, Vol. 20, 1984, pp. 175-181.

Instability of a Partially Delaminated Surface Layer of an Oscillating Cylinder

W. J. Bottega*
Rutgers University, Piscataway, New Jersey 08855

Introduction

THE subject of delamination buckling and growth has been the topic of many investigations over the past few years, primarily in the context of flat layers. Recently, however, several studies pertaining to cylindrical layers have been conducted.[1-7] (An extensive list of references to the field can be found in Refs. 3 and 4.) These studies have been concerned primarily with the buckling[1,2,7] and quasistatic growth[3-6] of thin-film delaminations under various loading and kinematical conditions. Aspects of the dynamics of the phenomena (both buckling and growth[8] and vibration about buckled states[9]) have been considered for specific configurations of flat layers as well. It appears that the case of buckling and growth induced by periodic loading, however, has not been addressed similarly.

Here, we consider the problem of a thin elastic layer adhered to the surface of a cylindrical substrate such that a "thru" delamination exists over a portion of the interface and the surface of the substrate oscillates uniformly. It shall be assumed for simplicity that the gap between the layer and the substrate in the damaged region is sufficiently deep to prevent contact for small deflections of the film. In what follows, the substrate is considered to be extremely stiff relative to the layer, such that its behavior is effectively unaltered by the presence of the layer. Thus, the small amplitude motion of the surface of the substrate is considered as prescribed and uniform.

The shallow arch theory employed as the mathematical model of the layer in the recent studies concerning delamination of cylindrical layers[4-6] shall be adopted here, and the corresponding equations of motion derived. Buckling, or instability, of the layer (i.e., the onset of large radial deflections of the layer measured with respect to the substrate) shall be

Received Jan. 8, 1990; revision received Feb. 12, 1990; accepted for publication Feb. 13, 1990.

*Associate Professor, Department of Mechanics and Materials Science. Member AIAA.